IYGB – MP1 PAPER H – QUESTION 1

MANIPULATE AS FOLLOWS

$$\frac{90}{\sqrt{3}} - \sqrt{6}\sqrt{8} - (2\sqrt{3})^3$$

$$= \frac{90\sqrt{3}}{\sqrt{3}\sqrt{3}} - (\sqrt{3}\underbrace{\sqrt{2}\times\sqrt{2}}_{2}\underbrace{\sqrt{2}\sqrt{2}}_{2}) - (2\sqrt{3}\times 2\sqrt{3}\times 2\sqrt{3})$$

3

$$= \frac{90\sqrt{3}}{3} - 2\times 2\times\sqrt{3} - 2\times2\times2\times3\times\sqrt{3}$$

$$= 30\sqrt{3} - 4\sqrt{3} - 24\sqrt{3}$$

$$= \underline{\underline{2\sqrt{3}}}$$

IYGB - MP1 PAPER H - QUESTION 2

a) USING THE STANDARD EXPANSION FORMULA

$$f(x) = (1-2x)^8 = 1 + \frac{8}{1}(-2x)^1 + \frac{8\times7}{1\times2}(-2x)^2 + \frac{8\times7\times6}{1\times2\times3}(-2x)^3 + \dots$$

$$(1-2x)^8 = 1 - 16x + 28(+4x^2) + 56(-8x^3) + \dots$$

$$(1-2x) = 1 - 16x + 112x^2 - 448x^3 + \dots$$

b) USING PART (a)

$$(2+3x)(1-2x)^8 = (2+3x)(1-16x+112x^2-448x^3+\dots)$$

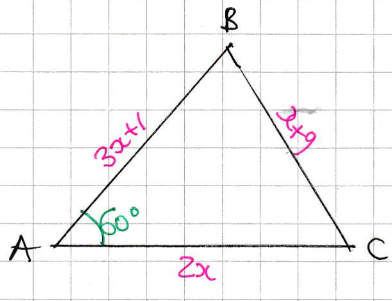
$$= 2 - 32x + 224x^2 - 896x^3 + \dots$$

$$\quad\;\; 3x - 48x^2 + 336x^3 + \dots$$

$$= 2 - 29x + 176x^2 - 560x^3 + \dots$$

IYGB - MP1 PAPER H - QUESTION 3

BY THE COSINE RULE WE HAVE



$$\Rightarrow |BC|^2 = |AB|^2 + |AC|^2 - 2|AB||AC|\cos 60$$

$$\Rightarrow (x+9)^2 = (3x+1)^2 + (2x)^2 - 2(3x+1)(2x) \times \frac{1}{2}$$

$$\Rightarrow (x+9)^2 = (3x+1)^2 + 4x^2 - 2x(3x+1)$$

$$\Rightarrow x^2 + 18x + 81 = 9x^2 + 6x + 1 + 4x^2 - 6x^2 - 2x$$

$$\Rightarrow 0 = 6x^2 - 14x - 80$$

$$\Rightarrow 0 = 3x^2 - 7x - 40$$

BY INSPECTION OR QUADRATIC FORMULA

$$\Rightarrow (3x+8)(x-5) = 0$$

$$\Rightarrow x = \begin{cases} 5 \\ \cancel{-\frac{8}{3}} \end{cases} \qquad x > 0$$

FINALLY THE AREA CAN BE FOUND

$$\Rightarrow \text{AREA} = \frac{1}{2}|AB||AC|\sin 60°$$

$$\Rightarrow \text{AREA} = \frac{1}{2}(3x+1)(2x) \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \text{AREA} = \frac{1}{2} \times 16 \times 10 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \text{AREA} = 40\sqrt{3}$$

IYGB – MP1 PAPER H – QUESTION 4

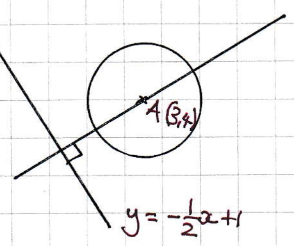
a) FIND THE CIRCLE PARTICULARS

$$\Rightarrow x^2+y^2-6x-8y+21=0$$
$$\Rightarrow x^2-6x+y^2-8y+21=0$$
$$\Rightarrow (x-3)^2-9+(y-4)^2-16+21=0$$
$$\Rightarrow (x-3)^2+(y-4)^2=4$$

CENTRE AT $(3,4)$ RADIUS 2

GRADIENT OF L

$$x+2y=2$$
$$2y=-x+2$$
$$y=-\frac{1}{2}x+1$$

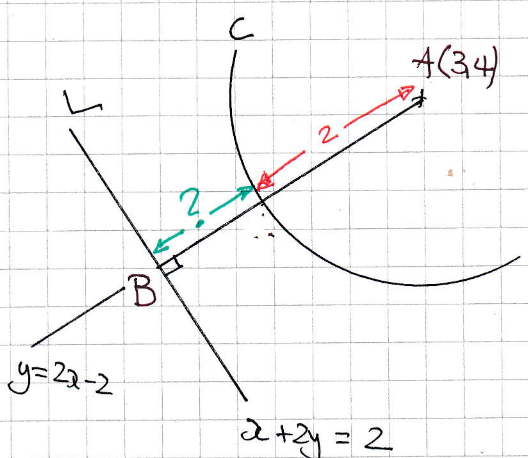


REQUIRED LINE HAS GRADIENT +2 & PASSES THROUGH $(3,4)$

$$y-y_0=m(x-x_0)$$
$$y-4=2(x-3)$$
$$y-4=2x-6$$
$$y=2x-2$$

b) SOLVING SIMULTANEOUSLY TO FIND THE INTERSECTION OF THE TWO LINES

$$\left.\begin{array}{l} y=2x-2 \\ x+2y=2 \end{array}\right\} \Rightarrow$$



$$x+2(2x-2)=2$$
$$x+4x-4=2$$
$$5x=6$$
$$x=\frac{6}{5}$$

IYGB - MP1 PAPER H - QUESTION 4

$$y = 2x - 2 = 2\left(\frac{6}{5}\right) - 2 = \frac{12}{5} - 2 = \frac{2}{5}$$

$$\therefore B\left(\frac{6}{5}, \frac{2}{5}\right)$$

DISTANCE AB, USING $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$A(3,4) \quad B\left(\frac{6}{5}, \frac{2}{5}\right)$

$$\Rightarrow |AB| = \sqrt{\left(3 - \frac{6}{5}\right)^2 + \left(4 - \frac{2}{5}\right)^2}$$

$$\Rightarrow |AB| = \sqrt{\frac{81}{25} + \frac{324}{25}}$$

$$\Rightarrow |AB| = \sqrt{\frac{81}{5}}$$

$$\Rightarrow |AB| = \frac{9}{5}\sqrt{5}$$

$\therefore$ REQUIRED DISTANCE IS $\frac{9}{5}\sqrt{5} - 2$

IYGB - MP1 PAPER H - QUESTION 5

PROCEED BY FORMING AN EQUATION BASED ON THE DISCRIMINANT

REPEATED ROOTS $\Rightarrow b^2-4ac=0$

$$\Rightarrow \underbrace{(p+5)^2}_{b^2} - 4\times \underbrace{3(p+2)}_{a}\times \underbrace{p}_{c} = 0$$

$$\Rightarrow p^2+10p+25 - 12p(p+2) = 0$$

$$\Rightarrow p^2+10p+25-12p^2-24p = 0$$

$$\Rightarrow -11p^2-14p+25=0$$

$$\Rightarrow 11p^2+14p-25=0$$

$$\Rightarrow (11p+25)(p-1)=0$$

$$\boxed{p = \begin{cases} 1 \\ -\frac{25}{11} \end{cases}}$$

EACH OF THESE TWO VALUES OF p, PRODUCES A QUADRATIC EQUATION IN x, WHICH MUST HAVE REPEATED ROOTS

- IF $p=1$

$3(p+2)x^2+(p+5)x+p=0$

$9x^2+6x+1=0$

$(3x+1)^2=0$

$x=-\frac{1}{3}$

- IF $p=-\frac{25}{11}$

$3(p+2)x^2+(p+5)x+p=0$

$-\frac{9}{11}x^2+\frac{30}{11}x-\frac{25}{11}=0$ $\quad \times 11$

$-9x^2+30x-25=0$ $\quad \times(-1)$

$9x^2-30x+25=0$

$(3x-5)^2=0$

$x=\frac{5}{3}$

IYGB - MP1 PAPER H - QUESTION 6

LOOKING AT THE DIAGRAM

$y = 4x - x^2$

$y = x(4-x)$

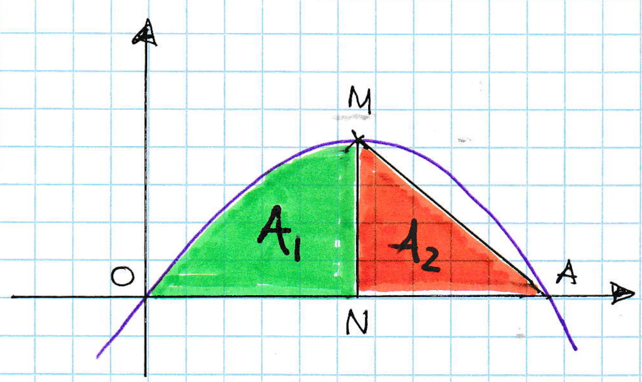
$\therefore$ $A(4,0)$ BY INSPECTION

$N(2,0)$ BY SYMMETRY

$M(2,4)$ $\Leftarrow$ $y = 4\times2 - 2^2$

$y = 8 - 4$

$y = 4$



AREA OF TRIANGLE $= A_2 = \frac{1}{2} \times |NA||MN|$

$= \frac{1}{2} \times 2 \times 4$

$= 4$

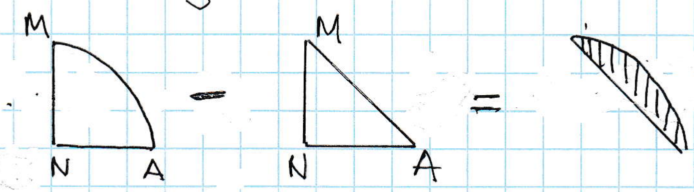
AREA UNDER CURVE $= A_1 = \int_0^2 4x - x^2 \, dx$

$= \left[2x^2 - \frac{1}{3}x^3\right]_0^2$

$= \left(8 - \frac{8}{3}\right) - 0$

$= \frac{16}{3}$

REQUIRED AREA =



SAME AS A_1

$= \frac{16}{3} - 4$

$= \frac{4}{3}$

IYGB – MP1 PAPER H – QUESTION 7

a) $f(x) = x^3 - 3x + 2$

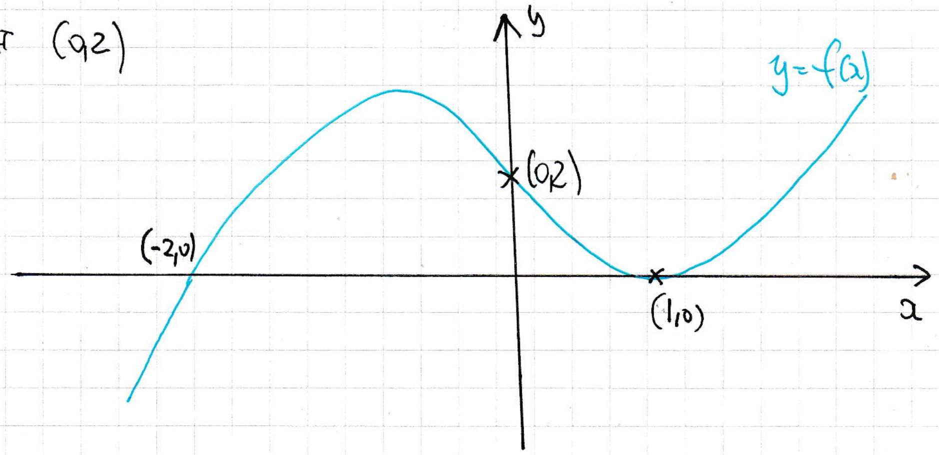
BY INSPECTION $x=1$ YIELDS ZERO, I.E $x-1$ IS A FACTOR

$$
\begin{array}{r|l}
 & x^2 + x - 2 \\
\hline
x-1 & x^3 \qquad -3x+2 \\
 & -x^3 + x^2 \\
\hline
 & \quad x^2 - 3x + 2 \\
 & \quad -x^2 + x \\
\hline
 & \qquad -2x + 2 \\
 & \qquad +2x - 2 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$$
\begin{aligned}
\therefore \quad f(x) &= (x-1)(x^2+x-2) \\
&= (x-1)(x-1)(x+2) \\
&= (x-1)^2(x+2)
\end{aligned}
$$

b) $f(x) = (x-1)^2(x+2)$

- TOUCHING POINT AT $(1,0)$
- CROSSING POINT AT $(-2,0)$
- y INTERCEPT AT $(0,2)$



IYGB – MP1 PAPER H – QUESTION 7

c) SOLVING THE EQUATION

$\Rightarrow f(x) = (x-1)^2$

$\Rightarrow (x-1)^2(x+2) = (x-1)^2$

$\Rightarrow (x-1)^2(x+2) - (x-1)^2 = 0$

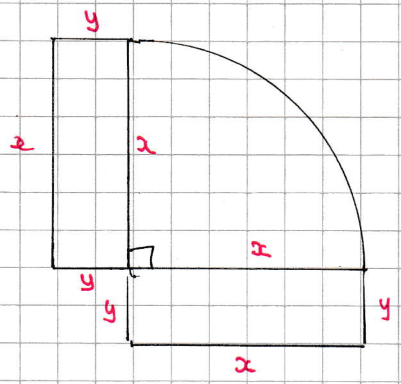
$\Rightarrow (x-1)^2\left[(x+2) - 1\right] = 0$

$\Rightarrow (x-1)^2(x+1) = 0$

$\Rightarrow x = \begin{cases} 1 \\ -1 \end{cases}$

IYGB - MP1 PAPER H - QUESTION 8

a)



CONSTRAINT ON AREA

$A = 12.25$

$2xy + \frac{1}{4} \times \pi x^2 = 12.25$ $\times 4$

$8xy + \pi x^2 = 49$

$\frac{8xy}{2x} + \frac{\pi x^2}{2x} = \frac{49}{2x}$ $\div 2x$

$4y + \frac{\pi}{2}x = \frac{49}{2x}$

$4y = \frac{49}{2x} - \frac{1}{2}\pi x$

PERIMETER $= 2x + 4y + \frac{1}{4}(2\pi x)$

$\Rightarrow P = 2x + 4y + \frac{1}{2}\pi x$

$\Rightarrow P = 2x + \left(\frac{49}{2x} - \cancel{\frac{1}{2}\pi x}\right) + \cancel{\frac{1}{2}\pi x}$

$\Rightarrow P = 2x + \frac{49}{2x}$ // AS REQUIRED

b) DIFFERENTIATE & SOLVE FOR ZERO

$\Rightarrow P = 2x + \frac{49}{2}x^{-1}$

$\Rightarrow \frac{dP}{dx} = 2 - \frac{49}{2}x^{-2}$

FOR MIN/MAX $\frac{dP}{dx} = 0$

$\Rightarrow 2 - \frac{49}{2x^2} = 0$

$\Rightarrow 2 = \frac{49}{2x^2}$

$\Rightarrow 4x^2 = 49$

$\Rightarrow x^2 = 12.25$

$\Rightarrow x = 3.5 \quad (x>0)$

IYGB – MP1 PAPER H – QUESTION 8

USE 2ND DERIVATIVE TO JUSTIFY MINIMUM.

$$\Rightarrow \frac{dP}{dx} = 2 - \frac{49}{2}x^{-2}$$

$$\Rightarrow \frac{d^2P}{dx^2} = 49x^{-3} = \frac{49}{x^3}$$

$$\Rightarrow \left.\frac{d^2P}{dx^2}\right|_{x=3.5} = \frac{8}{7} > 0$$

INDEED $x=3.5$ MINIMIZES P

c) USING THE CONSTRAINT EQUATION

$$\Rightarrow 8xy + \pi x^2 = 49$$

$$\Rightarrow 8(3.5)y + \pi(3.5)^2 = 49$$

$$\Rightarrow 28y + \frac{49}{4}\pi = 49$$

$\div 7$

$$\Rightarrow 4y + \frac{7}{4}\pi = 7$$

$\times 4$

$$\Rightarrow 16y + 7\pi = 28$$

$$\Rightarrow 16y = 28 - 7\pi$$

$$\Rightarrow y = \frac{7(4-\pi)}{16}$$

AS REQUIRED

IYGB - MP1 PAPER H - QUESTION 9

$$\boxed{f(x) = \ln 4x \quad x > 0}$$

TIDYING UP THE EQUATION

$$\Rightarrow f(x) + f(x^2) + f(x^3) = 6$$

$$\Rightarrow \ln 4x + \ln(4x^2) + \ln(4x^3) = 6$$

$$\Rightarrow \ln\left[4x \times 4x^2 \times 4x^3\right] = 6$$

$$\Rightarrow \ln(64x^6) = 6$$

$$\Rightarrow 64x^6 = e^6$$

$$\Rightarrow x^6 = \frac{e^6}{64}$$

$$\Rightarrow x^6 = \frac{e^6}{2^6}$$

$$\Rightarrow x^6 = \left(\tfrac{1}{2}e\right)^6$$

$$\Rightarrow \underline{x = +\tfrac{1}{2}e} \qquad x > 0$$

IYGB – MP1 PAPER H – QUESTION 10

$$\Longrightarrow \cos(4\psi - 120) = \cos 200$$

$$\Longrightarrow \begin{cases} 4\psi - 120 = 200 \pm 360n \\ 4\psi - 120 = (360 - 200) \pm 360n \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$$\Longrightarrow \begin{cases} 4\psi - 120 = 200 \pm 360n \\ 4\psi - 120 = 160 \pm 360n \end{cases}$$

$$\Longrightarrow \begin{cases} 4\psi = 320 \pm 360n \\ 4\psi = 280 \pm 360n \end{cases}$$

$$\Longrightarrow \begin{cases} \psi = 80 \pm 90n \\ \psi = 70 \pm 90n \end{cases}$$

IN THE RANGE GIVEN

$$\psi = 80°, 170°, 70°, 160°$$

IYGB - MP1 PAPER H - QUESTION 11

WORKING AS FOLLOWS

LET THE CONSECUTIVE EVEN POWERS OF 2 BE 2^{2n} & 2^{2n+2}

$$\Rightarrow \quad 2^{2n} + 2^{2n+2} = 2^{2n} + 2^{2n} \times 2^{2}$$
$$= 2^{2n} + 4 \times 2^{2n}$$
$$= 5 \times 2^{2n}$$
$$= 5 \times (2^{2})^{n}$$
$$= 5 \times 4^{n}$$

NOW 4^n IS A MULTIPLE OF 4, AS A POWER OF 4; SAY $4^n = 4k$ FOR SOME POSITIVE INTEGER k

$$\ldots = 5 \times 4k$$
$$= 20k$$

INDEED A MULTIPLE OF 20

IYGB - MP1 PAPER H - QUESTION 12

PROCEED AS FOLLOWS

$-\frac{1}{2} < x < \frac{7}{8}$

$-4 < 8x < 7$

$-\frac{1}{6} < y < \frac{2}{3}$

$-2 < 12y < 8$

$-12y > -8$ OR $-12y < +2$

$-8 < -12y < 2$

NOW WE HAVE

$T = 8x - 12y + 7 \Longrightarrow$

$$-4 < 8x < 7$$
$$-8 < -12y < 2$$

$$-12 < 8x - 12y < 9$$

$$-5 < 8x - 12y + 7 < 16$$

$$-5 < T < 16$$

IYGB - MP1 PAPER H - QUESTION 13

USING THE CO-ORDINATES TO SET SIMULTANEOUS EQUATIONS

$$(2,10) \Longrightarrow 10 = a \times 2^n$$
$$(6,100) \Longrightarrow 100 = a \times 6^n$$

DIVIDING THE EQUATIONS, SIDE BY SIDE

$$\frac{\cancel{a} \times 6^n}{\cancel{a} \times 2^n} = \frac{100}{10} \Longrightarrow \frac{6^n}{2^n} = 10$$
$$\Longrightarrow 3^n = 10$$
$$\Longrightarrow \log 3^n = \log 10$$
$$\Longrightarrow n \log 3 = 1$$
$$\Longrightarrow n = \frac{1}{\log 3} \approx 2.096$$

USING: $10 = a \times 2^n$

$$a = \frac{10}{2^n} = \frac{10}{2^{2.096...}} = 2.3392155...$$

$$\therefore \; a \simeq 2.339$$
$$n \simeq 2.096$$